Review

$(a = -\omega^2 x)$

$\omega = \frac{2\pi}{T} = (2\pi f)$

If $x = 0$, at $t = 0$

$x = x_0 \sin \omega t$

$v = v_0 \cos \omega t$

$(v = x_0 \omega \cos \omega t)$

$(a = -a_0 \sin \omega t)$

$(a = -x_0 \omega^2 \sin \omega t)$

$(a = -\omega^2 x)$

$v_0 = x_0 \omega$

If $x = x_0$, at $t = 0$

$x = x_0 \cos \omega t$

$v = -v_0 \sin \omega t$

$(v = -x_0 \omega \sin \omega t)$

$(a = -a_0 \cos \omega t)$

$(a = -x_0 \omega^2 \cos \omega t)$

$(a = -\omega^2 x)$

don't really need

$$v = \pm \omega \sqrt{(x_0^2 - x^2)}$$

EXAMPLE:

A pendulum has a period of 1.2 s and an amplitude of 0.10 m. Calculate the displacement, velocity, and acceleration of the pendulum bob 0.70 s after it is released.

T $\quad x_0$

Graph the displacement, velocity and acceleration vs time. (over 1 full period)

$$\omega = \frac{2\pi}{T}$$

$$\omega = \frac{2\pi}{1.2s}$$

$$\boxed{\omega = 5.2 s^{-1}}$$

$$x = x_0 \cos \omega t$$

$$\boxed{x = (0.10)\cos(5.2t)}$$

m

$$v = -x_0 \omega \sin \omega t$$

$$v = -(0.10)(5.2)\sin(5.2t)$$

$$\boxed{v = -(0.52)\sin(5.2t)}$$

ms^{-1}

reflection of sine

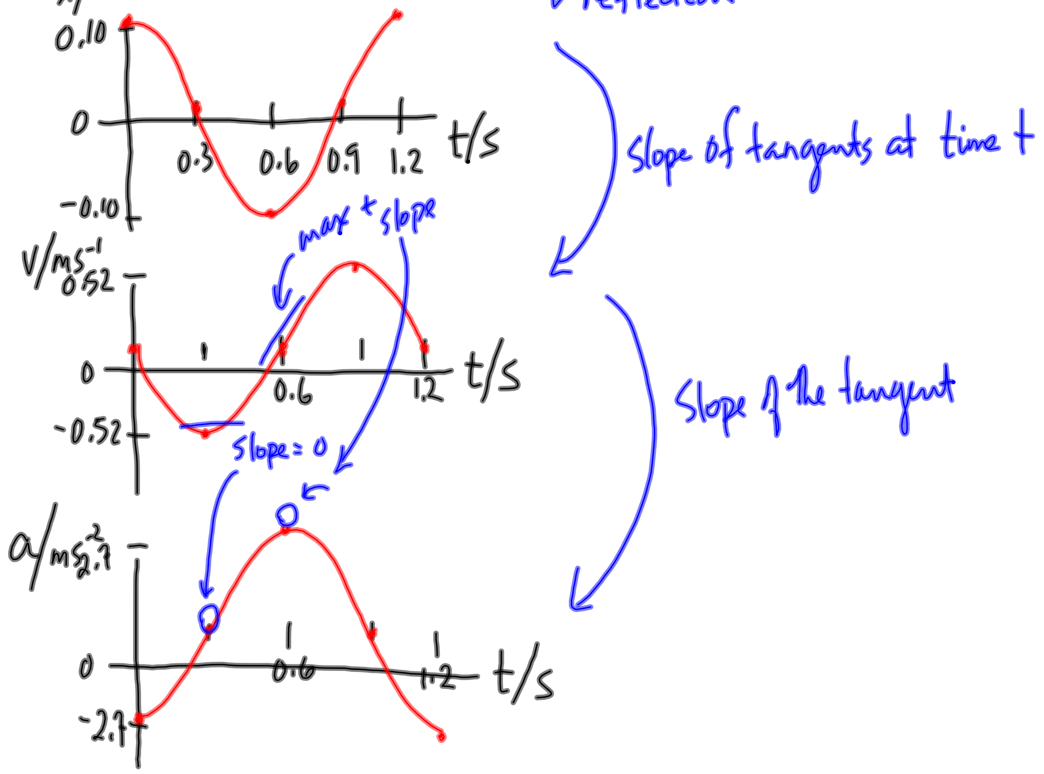
$$a = -x_0 \omega^2 \cos \omega t$$

$$a = -(0.10)(5.2)^2 \cos 5.2t$$

$$\boxed{a = -2.7\cos 5.2t}$$

ms^{-2}

reflection



Meaning of Phase + Phase Difference

Think of ωt and its units: $\text{rad}\,s^{-1}s$ = radians

So ωt can interpreted as an angle.

The phase of a body at an instant in time is the value ωt at that instant where $\omega = \frac{2\pi}{T}$ or $2\pi f$

Example:

phase difference

- When 2 bodies are oscillating, if one is $\frac{\pi}{2}$ ahead of the other in phase it means that it is a quarter of a period ahead of the other.
- If they were in opposite phase, then one is π ahead of the other in phase (or $\frac{1}{2}$ of the period).
- a difference of 2π (or increments of 2π) means that there is a delay in the start, but they are still in phase. (by 1 full period)

(or increments of the period)

$\omega = 2\pi f = 2\pi(2.5 s^{-1}) = 15.7 s^{-1}$

EXAMPLE:

A mass of 1.5 kg undergoes SHM with a frequency of 2.5 Hz and an amplitude of 0.50 m. x_0

a) What is the maximum restoring force on the body? $F = ma$

b) What is the magnitude of the restoring force when the mass is 0.25 m from its original position?

max acceleration ⇔ maximum displacement.

a) $F = ma$ + $a = -\omega^2 x$

$F = -m\omega^2 x$

$F = -(1.5 kg)(15.7 s^{-1})^2(0.50 m)$

$F = -185 N$

$F = -1.9 \times 10^2 N$

opposite the displacement (or towards the equilibrium)

The magnitude of the maximum force is $1.9 \times 10^2 N$

b) $F = -m\omega^2 x$

$F = -(1.5 kg)(15.7 s^{-1})^2(0.25 m)$

$\boxed{F = -92 N}$

92N towards the equilibrium position.

EXAMPLE:

A trolley held between two springs, when displaced, executes simple harmonic motion with a frequency of 2.0 Hz (f) and an amplitude of 4.0 cm (x_0).

a) Calculate the displacement, velocity, and acceleration of the trolley 0.30 s (t) after it passes through its equilibrium position. $x = x_0 \sin \omega t$

b) Calculate the maximum speed of the trolley. (at equilibrium (x=0))

c) Calculate the mass of the trolley. The force constant of the two springs, combined is 30 N m⁻¹ (k).

d) Sketch graphs of displacement versus time, velocity versus time, and acceleration versus time over one full cycle. Write the equation of each graph.

a) first find ω: $\omega = 2\pi f = 2\pi(2.0\,s^{-1}) = 12.6\,s^{-1}$

$x = x_0 \sin \omega t$

$x = (4.0\,cm)\sin\left(12.6\,s^{-1}(0.30\,s)\right)$

$x = -2.4\,cm$

$v = x_0 \omega \cos \omega t$

$v = (4.0\,cm)(12.6\,s^{-1})\cos\left(12.6\,s^{-1}(0.30\,s)\right)$

$v = -40\,cm\,s^{-1}$

$a = -x_0 \omega^2 \sin \omega t$

or

easier

$a = -\omega^2 x$

$a = -(12.6\,s^{-1})^2(-2.4\,cm)$

$a = +3.8 \times 10^2\,cm\,s^{-2}$

b) $v = \pm \omega \sqrt{(x_0^2 - x^2)}$

$v = \pm (12.6\,s^{-1})\sqrt{((4\,cm)^2 - (0\,cm)^2)}$

$v = \pm (12.6\,s^{-1})(4\,cm)$

$v = \pm 50\,cm\,s^{-1}$

speed $\Rightarrow$ $50\,cm\,s^{-1}$

or

$v_0 = x_0 \omega$

$v_0 = (4.0\,cm)(12.6\,s^{-1})$

$v_0 = 50\,cm\,s^{-1}$

c) Recall Hooke's Law: $F = -kx$

Recall Newton's Second Law: $F = ma$

The magnitudes are equal:

$kx = ma$

$m = \frac{kx}{a}$ need to match.

$m = \frac{(30\,N\,m^{-1})(0.024\,m)}{3.8\,m\,s^{-2}}$

$m = 0.19\,kg$